Longitudinal East-West Station Keeping (EWSK)

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1 Purpose of the analysis

This short note analyses the East-West perturbations for a geostationary satellite [R 1].

1.1 Generalities

Without any perturbation, starting at a longitude of 96.26° in a circular orbit 3 km below GEO, the orbital velocity is faster than at GEO, so the orbital period is smaller than an Earth sidereal day: it is clear that the longitude λ of the satcom with respect to the Earth rotating longitude, drifts naturally, it increases toward the east.

2 Data

The Earth equator is not a perfect circle for what concerns the gravity, but it can be approximated by an ellipse. The effect of this Earth equatorial ellipticity is a longitudinal drift acceleration $\ddot{\lambda}$. Its evolution versus the east longitude λ follows the next equation and is plotted hereunder [R 1].

$$\ddot{\lambda} = -0.00168 \sin 2(\lambda - \lambda_s) \quad \text{deg/day}^2 \qquad (2.91)$$

$$\ddot{\lambda} = -2.251\text{E-}13 \cdot \sin(2(\lambda - \lambda_s)) \quad \text{deg/s}^2$$

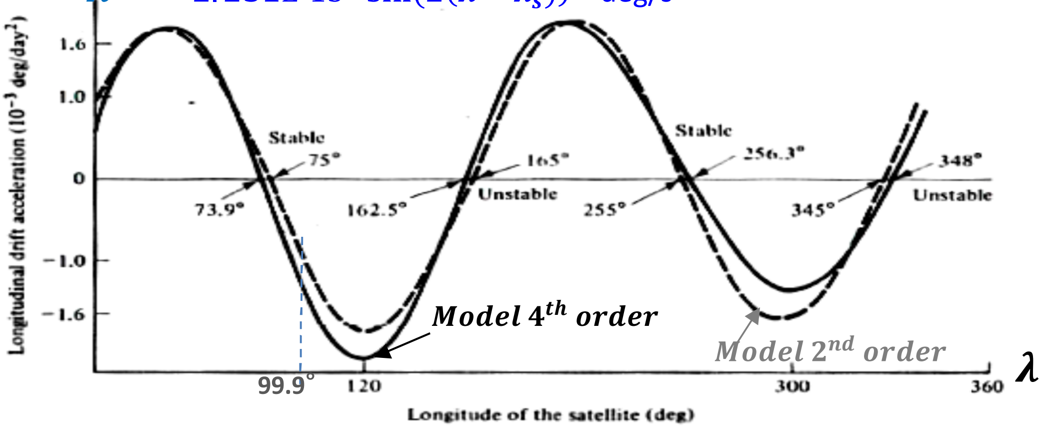
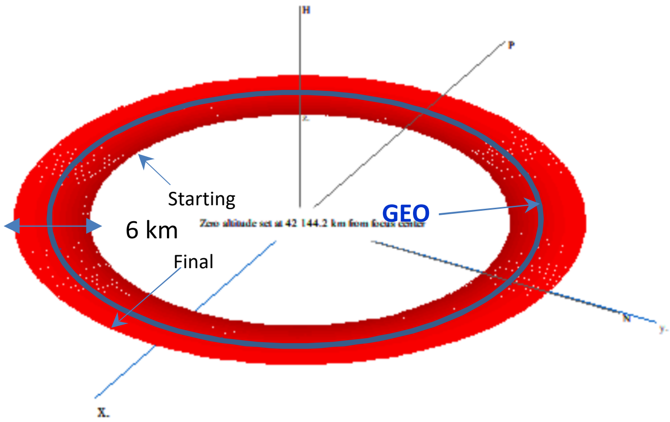


Figure 2.17 Longitudinal drift acceleration due to earth's ellipticity.

2.1 Continuous thrust simulating the longitude drift acceleration due to Earth ellipticity

On a satellite mass 5000 kg, using a constant thrust of 0.2 mN oriented along the orbital velocity and without any orbital perturbation, in 2.1 months one gets a spiral with the semi-major axis increasing linearly by 6 km (from GEO -3km up to GEO +3km, see the computed plot using [R 2] at right with a zero altitude set 20 km below GEO).



The orbital period of such so called "perturbed orbit" increases linearly with the time by 18 s.

The corresponding evolution of the east longitude of the satcom with respect to the Earth rotating longitude is shown in the next plots.

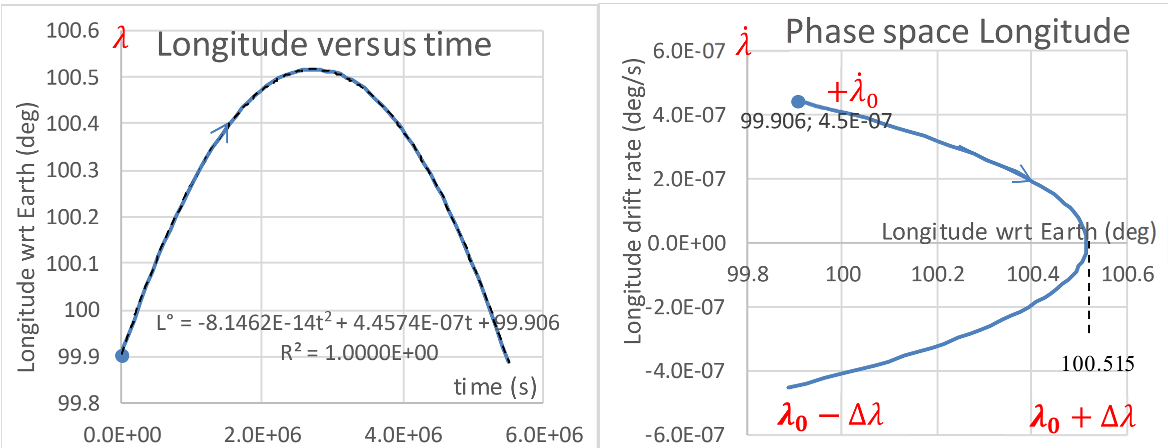


Figure 1 Case constant thrust along velocity, evolutions a) Longitude versus time b) drift rate starting at $+\dot{\lambda}_0$ versus Longitude

The phase space for the longitude shows that the drift rate $\dot{\lambda}$ is starting at $+\dot{\lambda}_0$ for a longitude of $\boldsymbol{\lambda_0} - \Delta\lambda$ is decreasing, hence the **drift acceleration is negative**. Note: this is not in contradiction with the fact that the thrust oriented along the velocity vector produces a **positive acceleration**, as it is well known that for orbital matters, acceleration means total energy increases but not for kinetic energy i.e. the orbital velocity decreases (cf. Vis viva equation $\frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$)

The evolution of the Longitude λ versus time provides by regression directly the second order equation:

$$\lambda = \frac{1}{2}\ddot{\lambda}t^2 + \dot{\lambda}_0 t + (\boldsymbol{\lambda_0} - \Delta\lambda) \qquad \text{(deg)}$$

Hence for this simulation, one finds:

$\ddot{\lambda} = -1.6292\text{E-}13$ deg/s² $\quad \dot{\lambda}_0 = 4.4574\text{E-}7$ deg/s and $\boldsymbol{\lambda_0} - \Delta\lambda = 99.906$ deg

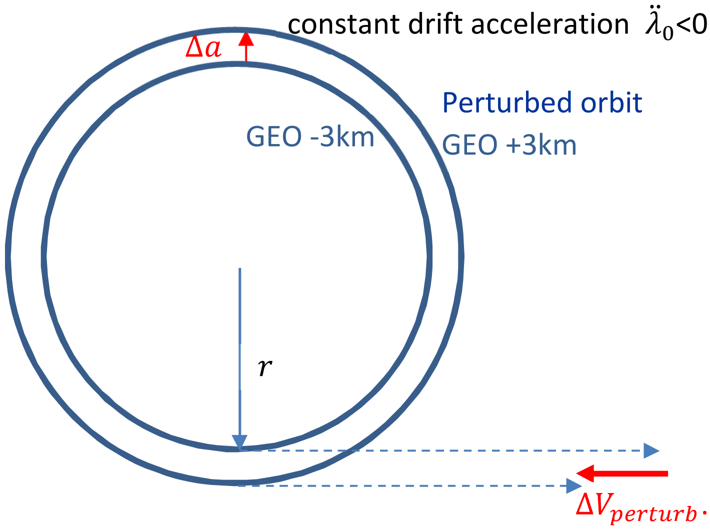
The ΔV produced by the continuous thrust of 0.2 mN during the 2.1 months of the spiral simulating the perturbation is : $\Delta V_{perturb} = 0.22$ m/s.

Note: the continuous positive acceleration of the satcom during the perturbation is $\frac{\boldsymbol{F}}{\boldsymbol{M}} = 4\text{E-}8$ m/s² which is also given by $\frac{\boldsymbol{F}}{\boldsymbol{M}} = \frac{-\boldsymbol{a}}{\boldsymbol{3}}\ddot{\boldsymbol{\lambda}}$ with $\ddot{\boldsymbol{\lambda}}$ in rd/s² and $\boldsymbol{a}$ the semi major axis 42 164 000 m.

3 Relation for the ΔV of the manoeuvre needed to correct the drift in longitude

3.1 Orbital variations

The effect of a constant drift acceleration $\ddot{\lambda}_0$<0 at the location of the satellite wrt rotating Earth, is shown at right, the semi-major axis a increases by Δa, here $\Delta a = +6km$.



According to Edelbaum relationship [R 3], the orbital velocity decreases by:

$\Delta V_{perturb} = V_{GEO-3km} - V_{GEO+3km}$.

3.2 Orbital correction $\boldsymbol{\Delta V}$ per year

The ΔV can be given again by the Edelbaum relationship, so identical to the one given by a continuous thrust of 0.2 mN during 2.1 months: that is, for $\ddot{\lambda} = -1.6292\text{E-}13$ deg/s² $\Delta V = 0.22$ m/s for one full cycle back to the starting orbit. For one full year, $\Delta V = 1.26$ m/s per year.

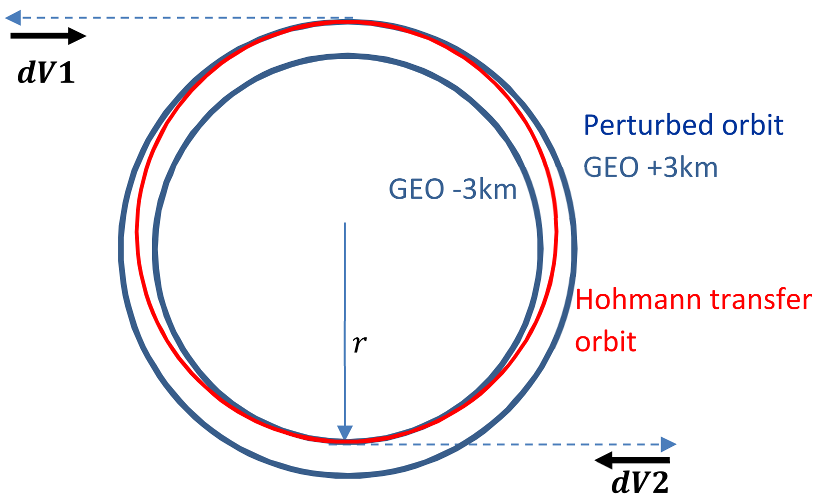
For other longitude λ , with $\lambda_s = 75°$, $\ddot{\lambda} = -2.251\text{E-}13 \sin(2(\lambda - \lambda_s))$ deg/s², the $\boldsymbol{\Delta V}$ in absolute value because $\boldsymbol{\Delta V}$ to be produced is always a positive quantity whatever the direction is:

$$\boldsymbol{\Delta V} = |-1.74 \sin(2(\lambda - \lambda_s))| \quad \text{m/s per year}$$

4 Other relationship for the ΔV of the manoeuvre

The vis viva equation $\frac{V^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2a}$ is used to set the ΔV of the manoeuvre needed for a correction of the drift in longitude, so with a semi-major axis variation Δa opposite to the previous one of §3.1.

This can be performed by two similar impulses (via an intermediate Hohman transfer orbit) to reduce the altitude of the final orbit (from GEO+3km to GEO-3km, here $\mathrm{d}a = -6km$). The impulsive thrusts $\mathbf{d}\boldsymbol{V}_i$ shall be in this case oriented against the orbital velocity.



Each of the $\mathbf{d}\boldsymbol{V}_i$ are colinear to $\boldsymbol{V}$, so no need to rely on any vector, analyses in mono-dimension are relevant. The $\mathbf{d}\boldsymbol{V}_i$ effect is the variation of semi-major axis by $\mathrm{d}a_i = \frac{\mathrm{d}a}{2}$. The radius r at the location of the satellite does not change during the impulsive manoeuvre so: $\frac{(V+\mathbf{d}V_i)^2}{2} - \frac{\mu}{r} = \frac{-\mu}{2(a+\mathrm{d}a_i)}$

$$\frac{V^2}{2} + \frac{2V\mathbf{d}\boldsymbol{V_i}}{2} - \frac{\mu}{r} = \frac{-\mu}{2a(1+\mathrm{d}a_i/a)} = \frac{-\mu(1-\mathrm{d}a_i/a)}{2a}$$

$$\frac{V^2}{2} + \frac{2V\mathbf{d}\boldsymbol{V_i}}{2} - \frac{\mu}{r} = \frac{-\mu}{2a} + \frac{\mu \mathrm{d}a_i}{2a^2}$$ hence, it remains: $V\mathbf{d}\boldsymbol{V_i} = \frac{\mu}{2a^2}\mathrm{d}a_i$

Because of small variations, this is valid as well as for $\boldsymbol{dV} = \boldsymbol{dV1} + \boldsymbol{dV2}$ still colinear to $\boldsymbol{V}$

$$V\mathbf{d}\boldsymbol{V} = \frac{\mu}{2a^2}\mathrm{d}a$$

4.1 Mean angular velocity and $\boldsymbol{\Delta V}$

The **mean** angular velocity of the orbit is $n = \frac{d\theta}{dt}$ where θ is the true anomaly or angle from the center for circular orbits. Note: the mean orbital velocity is $\overline{V} = a\frac{d\theta}{dt}$ i.e. $\overline{V} = n\, a$

For a GEO satcom, the **mean** angular velocity follows the Earth angular velocity. Hence, the longitude of a GEO satcom wrt the rotating Earth is a constant longitude. And a variation $\mathrm{d}n$ on the **mean** angular velocity is equal to the longitude drift rate variation $\mathrm{d}\dot{\lambda}$.

With T the orbital period, $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ $\quad n = \frac{2\pi}{T}$ $\quad n = \sqrt{\frac{\mu}{a^3}}$

By log derivatives, one gets: $\frac{\mathrm{d}n}{n} = \frac{-3}{2}\frac{\mathrm{d}a}{a}$ and using $V \approx \overline{V} = na$

$V\mathbf{d}\boldsymbol{V} = \frac{\mu}{2a^2}\mathrm{d}a$ $\quad na\mathbf{d}\boldsymbol{V} = \frac{\mu}{2a^2}\cdot\frac{-2\mathrm{a}}{3n}\mathrm{d}n$ $\quad \mathbf{d}\boldsymbol{V} = \frac{-\mu}{3n^2a^2}\mathrm{d}n$ $\quad \mathbf{d}\boldsymbol{V} = \frac{-\mu a^3}{3\mu a^2}\mathrm{d}n$

$$\mathbf{d}\boldsymbol{V} = \frac{-a}{3}\,\mathrm{d}n$$

4.2 Synthesis

Because the satcom $\mathrm{d}n$ is equal to the longitude drift rate $\mathrm{d}\dot{\lambda}$, it is easy to see that a full cycle correction requires an amplitude in longitude drift rate $\mathrm{d}\dot{\lambda}$ $=2\dot{\lambda}_0$ (from $-\dot{\lambda}_0$ to $+\dot{\lambda}_0$).

$\mathbf{d}\boldsymbol{V} = \frac{-2a}{3}\dot{\lambda}_0$ m/s per manoeuvres

With Y_s the number of seconds per year,$Y_s = 365 \times 86400$ (s/year) , the number of manoeuvres per year is $\frac{Y_s}{T_{Cycle}}$ where T_{Cycle} is the time spent for describing the full cycle.

For the case $\ddot{\lambda} < 0$ i.e. $\ddot{\lambda} = -|\ddot{\lambda}|$ the variation in longitude drift rate is from $+\dot{\lambda}_0$ to $-\dot{\lambda}_0$.

By $\ddot{\lambda} = \frac{d\dot{\lambda}}{dt}$ $\quad \int_0^{T_{Cycle}}(-|\ddot{\lambda}|)dt = \int_{+\dot{\lambda}_0}^{-\dot{\lambda}_0} d\dot{\lambda}$ $\quad -|\ddot{\lambda}|T_{Cycle}=-\dot{\lambda}_0 - (+\dot{\lambda}_0)$

$T_{Cycle} = \frac{2\dot{\lambda}_0}{|\ddot{\lambda}|}$ So, the number of manoeuvres per year is $\frac{Y_s}{2\dot{\lambda}_0}|\ddot{\lambda}|$

Eventually, $\mathrm{d}V = \frac{-2a}{3}\dot{\lambda}_0\cdot\frac{Y_s}{2\dot{\lambda}_0}|\ddot{\lambda}|$ $\quad \mathbf{d}\boldsymbol{V} = \frac{-a}{3}Y_s|\ddot{\lambda}|$ m/s per year.

- This quantity does not depend on the number of manoeuvres per year, neither on the drift rate, neither the longitude drift $\Delta\lambda$ and is proportional to the longitude drift acceleration.

With $\ddot{\lambda}_{max} = 0.00168$ deg/day² $\quad \ddot{\lambda}_{max} = \mathbf{2.251}\mathrm{E\text{-}13}$ deg/s² $\quad \ddot{\lambda}_{max} = 3.92789\mathrm{E}-15$ rd/s², one gets again, the $\mathbf{d}\boldsymbol{V}$ in absolute value because $\boldsymbol{\Delta V}$ to be produced is always a positive quantity whatever the direction is: $\boldsymbol{\Delta V} = |-1.74\ \mathbf{sin}(2(\boldsymbol{\lambda} - \boldsymbol{\lambda_s}))|$ m/s per year

5 Phase space characteristics

The starting point is a positive drift rate $+\dot{\lambda}_0$ at longitude $\boldsymbol{\lambda_0} - \Delta\lambda$. A constant drift acceleration $\ddot{\lambda}$<0 acts on the satcom. The drift rate $\dot{\lambda} = \frac{d\lambda}{dt}$ decreases down to zero due to the negative drift acceleration and this will occur at longitude said $\boldsymbol{\lambda_0} + \Delta\lambda$ (Note: the middle point $\boldsymbol{\lambda_0}$ is deduced after).

$$\ddot{\lambda} = \frac{d\dot{\lambda}}{dt} \qquad \ddot{\lambda} = \frac{d\dot{\lambda}}{d\lambda} \cdot \frac{d\lambda}{dt} \qquad \ddot{\lambda} = \frac{d\dot{\lambda}}{d\lambda} \cdot \dot{\lambda} \qquad \ddot{\lambda} d\lambda = \dot{\lambda} d\dot{\lambda}$$

$$\int_{\lambda_0-\Delta\lambda}^{\lambda_0+\Delta\lambda} \ddot{\lambda} d\lambda = \int_{+\dot{\lambda}_0}^{0} \dot{\lambda} d\dot{\lambda} \quad \text{so} \quad \ddot{\lambda}\big((\lambda_0 + \Delta\lambda) - (\lambda_0 - \Delta\lambda)\big) = \left(\frac{0^2}{2} - \frac{\dot{\lambda}_0{}^2}{2}\right)$$

$2\Delta\lambda\ddot{\lambda} = -\frac{\dot{\lambda}_0{}^2}{2}$ Because $\ddot{\lambda}$<0 on can write $2\Delta\lambda|\ddot{\lambda}| = +\frac{\dot{\lambda}_0{}^2}{2}$.

Hence the drift in longitude:

$$\Delta\lambda = \frac{\dot{\lambda}_0^2}{4|\ddot{\lambda}|}$$

the drift rate:

$$\dot{\lambda}_0 = \sqrt{4|\ddot{\lambda}|\Delta\lambda}$$

The middle point $\boldsymbol{\lambda_0}$ is thus deduced from the starting point $\boldsymbol{\lambda_0} - \Delta\lambda$:

$$\boldsymbol{\lambda_0} = (\boldsymbol{\lambda_0} - \Delta\lambda) + \frac{\dot{\lambda}_0^2}{4|\ddot{\lambda}|}$$

References:

[R 1] Brij N. Agrawal, Design of Geosynchronous spacecraft 1986
[R 2] KopooS, TriaXOrbitaL tool 1989-2021
[R 3] T. N. Edelbaum, Propulsion requirements for controllable satellites, ARS Journal, August 1961, pp. 1079–1089.

La mécanique orbitale est une discipline étrange... La première fois que vous la découvrez, vous ne comprenez rien.. La deuxième fois, vous pensez que vous comprenez, sauf un ou deux points.. La troisième fois, vous savez que vous ne comprenez plus rien, mais à ce niveau vous êtes tellement habitué que ça ne vous dérange plus. attribué à Arnold Sommerfeld pour la thermodynamique, vers 1940
